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treatment of the subject, and some of the famous mathematicians who have contributed directly or indirectly to the development of its mathematical treatment. He showed by means of tables and graphs the methods usually employed in handling correlated data, such as the length and weight of ears of corn. He also developed mathematically two of the most commonly used units of correlation; viz., the correlation ratio and the coefficient of correlation as defined by Karl Pearson.

5. Professor Myers defined hyperbolic functions both analytically and geometrically. He derived the identities between hyperbolic functions, showed their period and obtained the formulæ for their integration. Some of the numerous uses for hyperbolic functions were pointed out.

6. The problem may be stated as follows: A large pulley of radius b turns by means of a belt a smaller pulley of radius a. At a certain instant a radius of the larger pulley makes an angle with a radius of the smaller pulley. As the pulleys turn, in what position will the outer ends of the spoke be closest together? Professor White showed that the distance may be a maximum or a minimum when the radii are parallel or when the radii form an isosceles triangle with the line of centers.

7. Professor Harshbarger discussed briefly the growth of the present courses with the division of time brought about by changing high school curricula. The plan of college algebra, three hours, trigonometry, two hours, analytics, five hours, and calculus, five hours, was compared to two other plans. The first suggested college algebra, five hours, trigonometry, two hours, and analytics, three hours, for the freshman year, with five hours of calculus for the first half of the sophomore year. The second plan suggested algebra, three hours, and trigonometry, two hours, for the first semester with analytics, four hours, and calculus, six hours, during the next two semesters.

Discussion on this subject was participated in by Professor Ashton, Professor Mitchell, Professor Remick, Professor Stouffer, Professor Stratton, and Professor White.

E. B. STOUFFER, *Secretary-Treasurer.*

THE DECEMBER MEETING OF THE MARYLAND-VIRGINIA-DISTRICT OF COLUMBIA SECTION.

The eighth regular meeting of the Maryland-Virginia-District of Columbia Section of the Mathematical Association of America was held at the U. S. Naval Academy, Annapolis, Md., on Dec. 11, 1920. The meeting consisted of two sessions with Professor L. S. Hulburt presiding.

The attendance was forty-seven, including the following thirty-four members of the Association: O. S. Adams, J. J. Arnaud, R. N. Ashmun, Clara L. Bacon, Sarah Beall, A. A. Bennett, G. A. Bingley, C. C. Bramble, J. A. Bullard, P. Capron, G. R. Clements, A. Cohen, G. H. Cresse, F. W. Darling, L. S. Dederick,

A. Dillingham, H. English, J. B. Eppes, A. Hall, W. M. Hamilton, W. E. Heal, P. E. Hemke, L. S. Hulburt, L. S. Johnston, W. D. Lambert, L. N. Morscher, F. D. Murnaghan, J. R. Musselman, C. H. Rawlins, Jr., H. M. Robert, Jr., R. E. Root, J. B. Scarborough, W. F. Shenton, C. A. Shook.

A generous luncheon was served to the members and their guests by the mathematical staff of the Naval Academy. A preliminary report of the committee appointed to cooperate with the National Committee on Mathematical Requirements was presented by Professor A. Cohen. The next meeting will be held in Washington, probably early in May.

The following papers were presented:

(1) "Some mechanical curiosities connected with the earth's field of force" by Mr. W. D. LAMBERT, U. S. Coast and Geodetic Survey.

(2) "Parallel components in statical equilibrium" by Professor G. R. CLEMENTS, U. S. Naval Academy.

(3) "Singular curves of a plane pencil" by Professor C. C. BRAMBLE, U. S. Naval Academy.

(4) "Remarks on a problem in geometry" by Dr. F. D. MURNAGHAN, Johns Hopkins University.

(5) "Some arithmetic operations with transfinite ordinals" by Professor A. A. BENNETT, Technical Staff, Army Ordnance.

(6) "Multiple improper integrals" by Mr. C. A. SHOOK, U. S. Naval Academy.

(7) "On a certain statistical problem" by Dr. J. R. MUSSELMAN, Johns Hopkins University.

Abstracts of the papers numbered in accordance with the above list of titles are given below.

1. By the earth's field of force, in Mr. Lambert's paper, was meant the field due to mass-attraction according to the Newtonian law combined with the centrifugal force of the earth's rotation. These forces are adequate in problems of equilibrium relative to the earth. As approximations to the form of the equipotential or level surfaces of the earth's field we frequently take parallel planes or concentric spheres. The limiting level surface of the earth's field is, however, not even spheroidal, but has an edge around the equator. The level surfaces lying near the earth are ellipsoids, the ellipticity of which increases with the major axis. The fact that they are not concentric spheres gives rise to what seem like paradoxes. Some of these paradoxes were examined and a geological application suggested for one of them. The peculiarities of the earth's field may be examined experimentally by the Eötvös torsion balance. The paper gave a brief description of the instrument and some account of the work done with it in determining the curvature of the geoid and adjacent level surfaces.

2. Professor Clements suggested a more explicit formulation of the conditions under which one system of parallel forces may be replaced by another which shall be called equivalent to it, than is found in many texts on elementary mechanics. By examples he pointed out that in computing the reactions at the pins of a

framed structure, considerable simplification can be had by replacing the load (either concentrated or distributed) on each member by an equivalent system of parallel forces acting at its points of connection with the structure. In particular, any bar having only two such points of connection becomes, for the purpose of computing the reactions at the pins, a 'two force piece' and may be 'cut' in the usual manner of the method of sections.

3. The number of curves of a plane pencil with a double point is $3(n-1)^2$ provided that the base points of the pencil are ordinary points. If all of the curves have an r-fold point at a base point, Professor Bramble showed that the number of curves which have a further double point is $(r-1)(3r+1)$ less than $3(n-1)^2$. This is shown by considering the behavior of the curves $F_1\phi_3 - F_3\phi_1 = 0$ and $F_2\phi_3 - F_3\phi_2 = 0$ at the r-fold point when $F_i = \partial F/\partial x_i$, $\phi_i = \partial\phi/\partial x_i$, F and ϕ being the base curves of the pencil.

4. Dr. Murnaghan discussed Darboux's problem of the cyclic quadrilateral (cf. *Nouvelles Annales de Mathématiques*, 1868, p. 138.) The quartic equation $a_0x^4 + 4a_1x^3 + 6a_2x^2 + 4a_3x + a_4 = 0$ with real or complex coefficients has four roots in the complex plane which are either cyclic or anticyclic (i.e., two lie on an Apollonian circle of the other two) provided that $A \equiv g_2^3/g_3^2$ is *real*; g_2 and g_3 being the two invariants of the quartic. When the points are cyclic the double-ratio of the four points is real and $(g_2^3/g_3^2) \geqq 27$, the equality holding only when two of the points coincide. In the anticyclic case the double ratio has modulus unity and $(g_2^3/g_3^2) \leqq 27$. When the coefficients are real, the anticyclic case is that when two roots are real and the others conjugate imaginaries; the cyclic case is that when all roots are real or all imaginary. If three of the roots are regarded as fixed and the fourth variable, parts of the geometry of the triangle may be interestingly discussed (thus the Apollonian circle is the locus of points whose pedal triangle is isosceles, etc.); $A = \bar{A}$ is the equation of the circumcircle and the three Apollonian circles. The center of the circumcircle is tied up with the other three centres.

5. In this paper, Professor Bennett noted that as a consequence of the uniqueness of left hand subtraction among Cantor ordinals, the highest common left-hand divisor of two ordinals may always be found by Euclid's algorithm. The system of ordinals is then extended by right-hand subtraction so that among the formal pairs thus resulting, addition, subtraction and multiplication are always uniquely possible.

6. In his paper Mr. Shook first defined uniform convergence of improper definite integrals. He then discussed a double integral, in which the upper limits are infinite and the integrand does not remain finite, giving a set of sufficient conditions for changing the order of integration. Finally he gave an application to a certain double integral which arises in the theory of beta and gamma functions.

7. An urn contains n balls numbered successively from 1 to n; the balls are drawn out one by one and the order of their appearance noted. Dr. Musselman derived a formula for the average error resulting from the drawings varying from

some given assigned arrangement. This result can be used for determining statistically whether stock judging is more than a matter of chance.

O. S. ADAMS, *Secretary-Treasurer.*

ON CERTAIN PROPERTIES OF MAKEHAM'S LAWS OF MORTALITY WITH APPLICATIONS.[1]

By H. L. RIETZ, University of Iowa.

1. Introduction. Early in the development of the mathematics of life insurance, the calculations became laborious. This situation led to efforts to discover laws of mortality from which monetary values for life insurance could be calculated. As early as 1725, De Moivre in his *Annuities upon Lives,* put forth his famous hypothesis that the decrements d_x of the number living l_x at age x in a mortality table is constant for all values of x. Very little data were available to verify this hypothesis, but even the data of that time should have made it appear that this assumption could at most be a rough approximation. The De Moivre hypothesis is, of course, equivalent to a statement that the number living l_x at age x can be given by a linear function of x.

Just about one hundred years later in the *Philosophical Transactions* of 1825,[2] Gompertz published the so called law of mortality which bears his name. Before discussing the hypothesis of Gompertz, it is desirable to explain the meaning of the expression "force of mortality." This expression is in common use by actuaries at the present time in connection with the doctrines of Gompertz and in other connections. It should perhaps be stated that Gompertz did not use the expression "force of mortality" in his papers, but used instead the expression "intensity of mortality" which has no technical meaning at present.

The "force of mortality" μ_x at any age x may be defined as the rate of change of the average number living l_x of an indefinitely large class of persons of age in the neighborhood of x, per individual of the group l_x. That is to say, if the function l_x has a derivative with regard to x,

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx},$$

the negative sign being used so that μ_x is positive.

In other words, we conceive a continuous function l_x which is proportional to the average number living at age x, out of an indefinitely large group of lives of age in the neighborhood of x. Then the force of mortality is defined as the derivative of this function with regard to age or time divided by l_x. Obviously

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[1] Read under slightly different title before the Mathematical Association of America, September 6, 1920.

[2] "On the nature of the function expressive of the law of human mortality and a new mode of determining the value of life contingencies," pp. 513–583.